SYSTEM OF FORCES

INTRODUCTION

The force is an important factor in the field of Mechanics, which may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion. e.g., a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump. In this case, the force is supplied by the muscular power of our arms and shoulders.

EFFECTS OF A FORCE

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion.

And if the body is already in motion, the force may accelerate it.

2. It may retard the motion of a body.

3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.

4. It may give rise to the internal stresses in the body, on which it acts.

CHARACTERISTICS OF A FORCE

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

1. Magnitude of the force (i.e. 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The direction of the line, along which the force acts (i.e. along OX, OY, at any angle from OX or OY etc.). It is also known as line of action of the force.
3. Nature of the force (i.e. whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.
4. The point at which (or through which) the force acts on the body.

PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

It states, "If a number of forces are simultaneously acting on a *particle, then the resultant of these forces will have the same effect as produced by all the forces".

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body".

VARIOUS SYSTEM OF FORCES & RESULTANT FORCE

When two or more forces act on a body, they are called to form a system of forces. Various system of forces are as given:

1. Coplanar forces: The forces, whose lines of action lie on the same plane, are known as coplanar forces.

2. Collinear forces: The forces, whose lines of action lie on the same line, are known as collinear forces.

3. Concurrent forces: The forces, which meet at one point, are known as concurrent forces.
The concurrent forces may or may not be collinear.

4. Coplanar concurrent forces: The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.

5. Coplanar non-concurrent forces: The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

6. Non-coplanar concurrent forces: The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

7. Non-coplanar non-concurrent forces: The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

RESULTANT FORCE

If a number of forces P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would produce the same effect as produced by all the given forces. This single force is called resultant force and the given forces P, Q, R ... etc. are called component forces.

COMPOSITION OF FORCES

The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

ANALYTICAL METHOD FOR RESULTANT FORCE

The resultant force, of a given system of forces, may be found out analytically by the following methods :

1. Parallelogram law of forces
2. Method of resolution.

PARALLELOGRAM LAW OF FORCES

It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

and $$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

Note. It the angle (α) which the resultant force makes with the other force F_2.

then $$\tan\alpha = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

Cor.

1. If $\theta = 0$ *i.e.*, when the forces act along the same line, then

$$R = F_1 + F_2 \qquad \text{...(Since } \cos 0° = 1\text{)}$$

2. If $\theta = 90°$ *i.e.*, when the forces act at right angle, then

$$\theta = R = \sqrt{F_1^2 + F_2^2} \qquad \text{...(Since } \cos 90° = 0\text{)}$$

3. If $\theta = 180°$ *i.e.*, when the forces act along the same straight line but in opposite directions, then $R = F_1 - F_2$ ...(Since $\cos 180° = -1$)

In this case, the resultant force will act in the direction of the greater force.

4. If the two forces are equal *i.e.*, when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta} = \sqrt{2F^2(1 + \cos\theta)}$$

$$= \sqrt{2F^2 \times 2\cos^2\left(\frac{\theta}{2}\right)} \qquad \text{...}\left[\because 1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$

$$= \sqrt{4F^2 \cos^2\left(\frac{\theta}{2}\right)} = 2F\cos\left(\frac{\theta}{2}\right)$$

NUMERICALS AND PRINCIPLE OF RESOLUTION

Example . *Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?*

Solution. Given : First force $(F_1) = 100$ N; Second force $(F_2) = 150$ N and angle between F_1 and F_2 $(\theta) = 45°$.

We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45°} \text{ N}$$

$$= \sqrt{10\,000 + 22\,500 + (30\,000 \times 0.707)} \text{ N}$$

$$= 232 \text{ N} \quad \textbf{Ans.}$$

Example . *Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.*

Solution. Given : Angle between the forces $\angle AOC = 120°$. Bigger force $(F_1) = 40$ N and angle between the resultant and F_2 $(\angle BOC) = 90°$;

Let F_2 = Smaller force in N

From the geometry of the figure, we find that $\angle AOB$,

$$\alpha = 120° - 90° = 30°$$

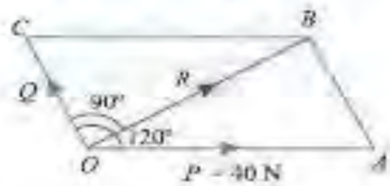


Fig. 2.1.

We know that

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°} = \frac{F_2 \sin 60°}{40 + F_2(-\cos 60°)}$$

$$\therefore \quad 0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866 F_2}{40 - 0.5 F_2}$$

$$40 - 0.5 F_2 = \frac{0.866 F_2}{0.577} = 1.5 F_2$$

$$\therefore \quad 2F_2 = 40 \quad \text{or} \quad F_2 = 20 \quad \textbf{Ans.}$$

RESOLUTION OF A FORCE

The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

PRINCIPLE OF RESOLUTION

It states, "*The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.*"

Example . *A machine component 1.5 m long and weight 1000 N is supported by two ropes AB and CD as shown in Fig. 2.2 given below.*

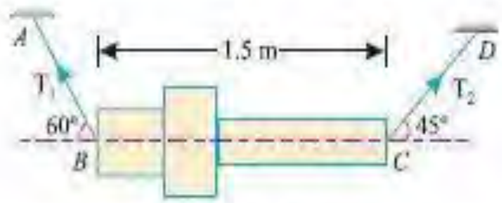


Fig. 2.2.

Calculate the tensions T_1 and T_2 in the ropes AB and CD.

Solution. Given : Weight of the component = 1000 N

Resolving the forces horizontally (*i.e.*, along *BC*) and equating the same,

$$T_1 \cos 60° = T_2 \cos 45°$$

$$\therefore \quad T_1 = \frac{\cos 45°}{\cos 60°} \times T_2 = \frac{0.707}{0.5} \times T_2 = 1.414\, T_2 \qquad ...(i)$$

and now resolving the forces vertically,

$$T_1 \sin 60° + T_2 \sin 45° = 1000$$

$$(1.414\, T_2)\, 0.866 + T_2 \times 0.707 = 1000$$

$$1.93\, T_2 = 1000$$

$$\therefore \quad T_2 = \frac{1000}{1.93} = 518.1 \text{N} \quad \textbf{Ans.}$$

and $$T_1 = 1.414 \times 518.1 = 732.6 \text{ N} \quad \textbf{Ans.}$$

METHOD OF RESOLUTION FOR THE RESULTANT FORCE

1. Resolve all the forces horizontally and find the algebraic sum of all the horizontal components (*i.e.*, $\sum H$).
2. Resolve all the forces vertically and find the algebraic sum of all the vertical components (*i.e.*, $\sum V$).
3. The resultant *R* of the given forces will be given by the equation :

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

4. The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan \theta = \frac{\sum V}{\sum H}$$

Example *The following forces act at a point :*

(i) *20 N inclined at 30° towards North of East,*

(ii) *25 N towards North,*

(iii) *30 N towards North West, and*

(iv) *35 N inclined at 40° towards South of West.*

Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Fig. 2.6.

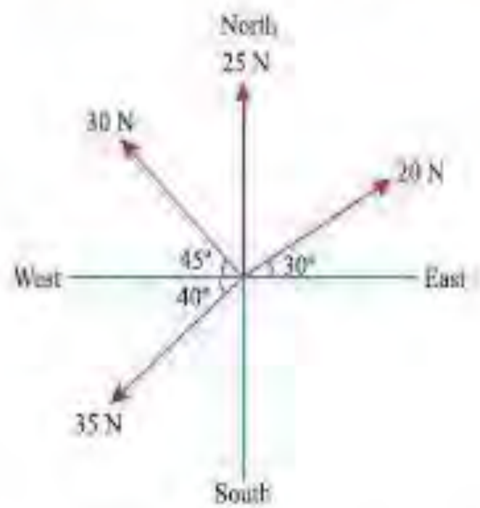


Fig. 2.6.

Magnitude of the resultant force

Resolving all the forces horizontally *i.e.*, along East-West line,

$$\Sigma H = 20 \cos 30° + 25 \cos 90° + 30 \cos 135° + 35 \cos 220° \text{ N}$$
$$= (20 \times 0.866) + (25 \times 0) + 30\,(-0.707) + 35\,(-0.766) \text{ N}$$
$$= -30.7 \text{ N} \qquad ...(i)$$

and now resolving all the forces vertically *i.e.*, along North-South line,

$$\Sigma V = 20 \sin 30° + 25 \sin 90° + 30 \sin 135° + 35 \sin 220° \text{ N}$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-0.6428) \text{ N}$$
$$= 33.7 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{33.7}{-30.7} = -1.098 \quad \text{or} \quad \theta = 47.7°$$

Since ΣH is negative and ΣV is positive, therefore resultant lies between 90° and 180°. Thus actual angle of the resultant = 180° − 47.7° = 132.3° **Ans.**

EQUILIBRIUM OF RIGID BODY

LAWS FOR THE RESULTANT FORCE

The resultant force, of a given system of forces, may also be found out by the following laws :

1. Triangle law of forces. **2.** Polygon law of forces.

TRIANGLE LAW OF FORCES

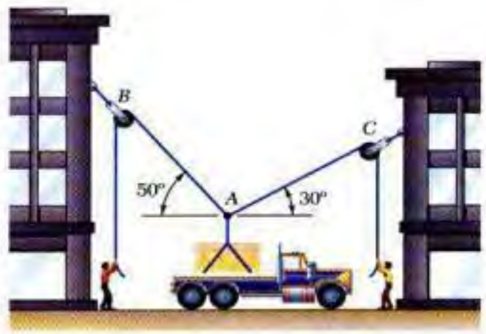
It states, "*If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.*"

POLYGON LAW OF FORCES

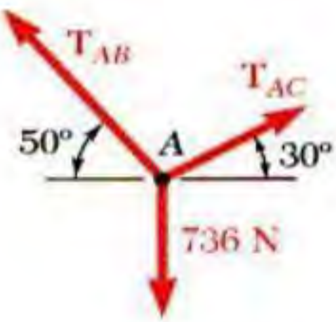
It is an extension of Triangle Law of Forces for more than two forces, which states, "*If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.*"

FREE BODY DIAGRAM

A free body diagram (FBD) is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a body in a given condition. They depict a body or connected bodies with all the applied forces and moments, and reactions, which act on the body. The body may consist of multiple internal members (such as a truss), or be a compact body (such as a beam). A series of free bodies and other diagrams may be necessary to solve complex problems.



Space Diagram: A sketch showing the physical conditions of the problem.

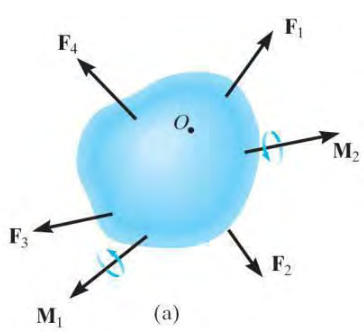


Free-Body Diagram: A sketch showing only the forces on the selected particle.

RIGID BODY EQUILIBRIUM

A rigid body will remain in equilibrium provided

- Sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

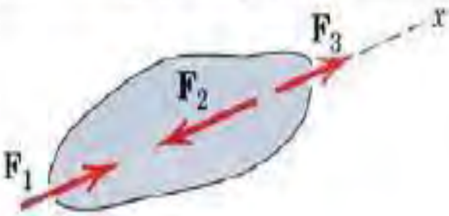
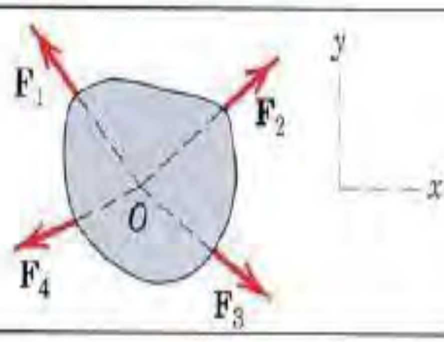
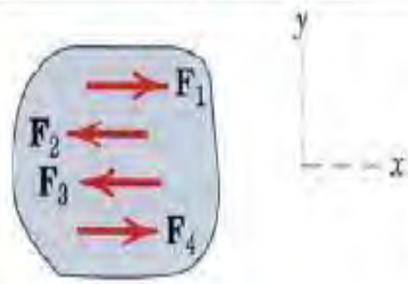
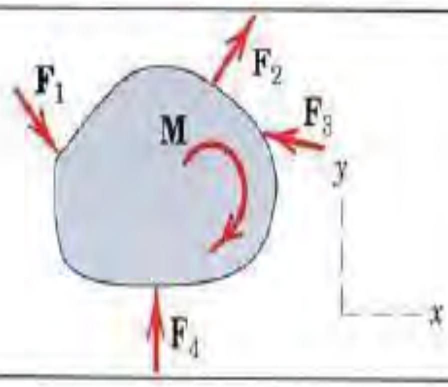


$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$

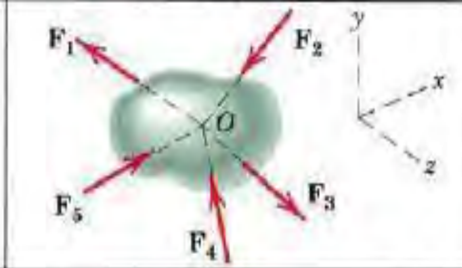
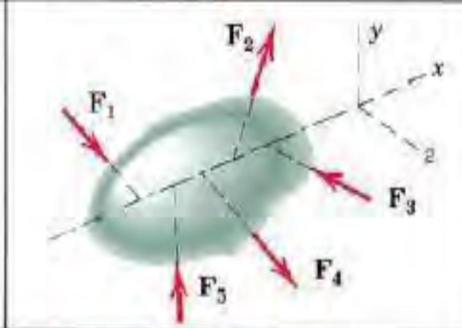
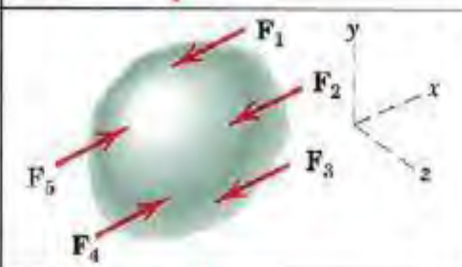
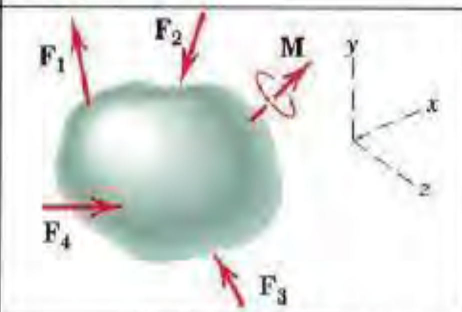
Rigid Body Equilibrium

Categories in 2-D

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Rigid Body Equilibrium

Categories in 3-D

<table>
<tr><th colspan="3">CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS</th></tr>
<tr><th>Force System</th><th>Free-Body Diagram</th><th>Independent Equations</th></tr>
<tr><td>1. Concurrent at a point</td><td>
</td><td>$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma F_z = 0$</td></tr>
<tr><td>2. Concurrent with a line</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_y = 0$
$\Sigma F_y = 0$ $\Sigma M_z = 0$
$\Sigma F_z = 0$</td></tr>
<tr><td>3. Parallel</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_y = 0$
$\Sigma M_z = 0$</td></tr>
<tr><td>4. General</td><td>
</td><td>$\Sigma F_x = 0$ $\Sigma M_x = 0$
$\Sigma F_y = 0$ $\Sigma M_y = 0$
$\Sigma F_z = 0$ $\Sigma M_z = 0$</td></tr>
</table>

SIR CHHOTU RAM INSTITUTE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

ENGINEERING MECHANICS

<u>TOPIC: SUPPORT REACTIONS</u>

see that the total weight of the fan and girder is acting through the supports of the girder on the walls. It is thus obvious, that walls must exert equal and upward reactions at the supports to maintain the equilibrium. The upward reactions, offered by the walls, are known as *support reactions.* As a mater of fact, the support reaction depends upon the type of loading and the support.

9.2. Types of Loading

Though there are many types of loading, yet the following are important from the subject point of view :

1. Concentrated or point load,
2. Uniformly distributed load,
3. Uniformly varying load.

9.3. Concentrated or Point Load

A load, acting at a point on a beam is known as a *concentrated or a point load* as shown in Fig. 9.1.

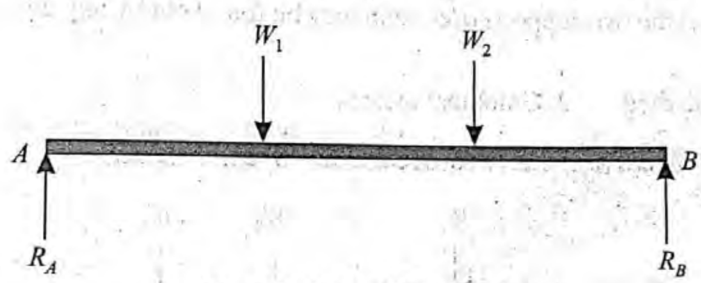


Fig. 9.1. Concentrated load.

In actual practice, it is not possible to apply a load at a point (*i.e.*, at a mathematical point), as it must have some contact area. But this area being so small, in comparison with the length of the beam, is negligible.

9.4. Uniformly Distributed Load

A load, which is spread over a beam, in such a manner that each unit length is loaded to the same extent, is known as *uniformly distributed load* (briefly written as U.D.L.) as shown in Fig. 9.2

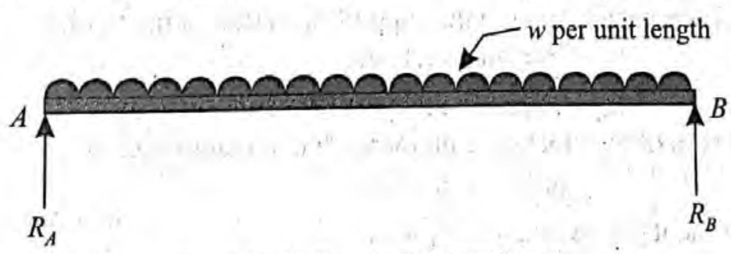


Fig. 9.2. Uniformly distributed load.

The total uniformly distributed load is assumed to act at the centre of gravity of the load for all sorts of calculations.

9.5. Uniformly Varying Load

A load, which is spread over a beam, in such a manner that its extent varies uniformly on each unit length (say from w_1 per unit length at one support to w_2 per unit length at the other support) is known as *uniformly varying load* as shown in Fig. 9.3.

Sometimes, the load varies from zero at one support to w at the other. Such a load is also called triangular load.

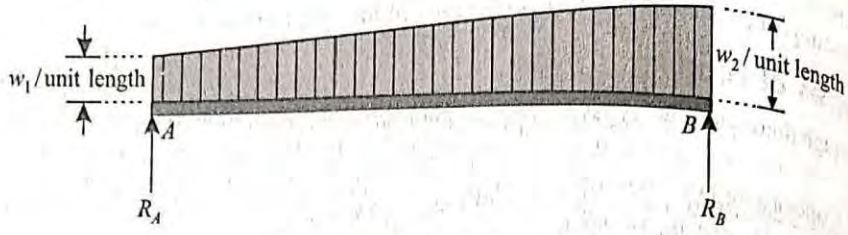


Fig. 9.3. Uniformly varying load.

Note : A beam may carry any one of the above-mentioned load system, or a combinations of the two or more.

9.6. Methods for the Reactions of a Beam

The reactions at the two supports of a beam may be found out by any one of the following two methods:

1. Analytical method 2. Graphical method.

9.7. Analytical Method for the Reactions of a Beam

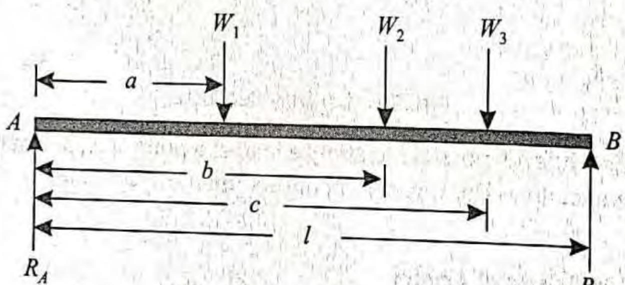


Fig. 9.4. Reactions of a beam.

Consider a *simply supported beam AB of span l, subjected to point loads W_1, W_2 and W_3 at distances of a, b and c, respectively from the support A, as shown in Fig. 9.4

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that sum of the clockwise moments due to loads about A

$$= W_1a + W_2b + W_3c \quad ...(i)$$

and anticlockwise moment due to reaction R_B about A

$$= R_B\, l \quad ...(ii)$$

Now equating clockwise moments and anticlockwise moments about A,

$$R_B\, l = W_1\, a + W_2\, b + W_3\, c \quad ...(\because \Sigma M = 0)$$

or

$$R_B = \frac{W_1 a + W_2 b + W_3 c}{l} \quad ...(iii)$$

Since the beam is in equilibrium, therefore

$$R_A + R_B = W_1 + W_2 + W_3 \quad ...(\because \Sigma V = 0)$$

and

$$R_A = (W_1 + W_2 + W_3) - R_B$$

* It will also be discussed in Art. 12.12

9.12. Simply Supported Beams

Fig. 9.6. Simply supported beam

It is a theoretical case, in which the end of a beam is simply supported over one of its support.

In such a case the reaction is always *vertical* as shown in Fig. 9.6.

Example 9.1. *A simply supported beam AB of span 5 m is loaded as shown in Fig. 9.7. Find the reactions at A and B.*

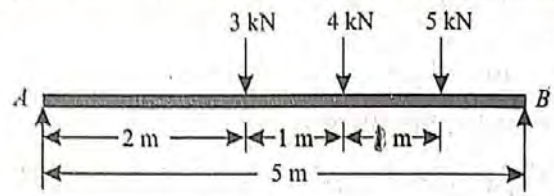


Fig. 9.7.

Solution. Given: Span (l) = 5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve analytically only.

We know that anticlockwise moment due to R_B about A

$$= R_B \times l = R_B \times 5 = 5\,R_B \text{ kN-m} \quad ...(i)$$

and sum of the clockwise moments about A,

$$= (3 \times 2) + (4 \times 3) + (5 \times 4) = 38 \text{ kN-m} \quad ...(ii)$$

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$5\,R_B = 38$$

or $R_B = \dfrac{38}{5} = 7.6$ kN **Ans.**

and $R_A = (3 + 4 + 5) - 7.6 = 4.4$ kN **Ans.**

Example 9.2. *A simply supported beam, AB of span 6 m is loaded as shown in Fig.9.8.*

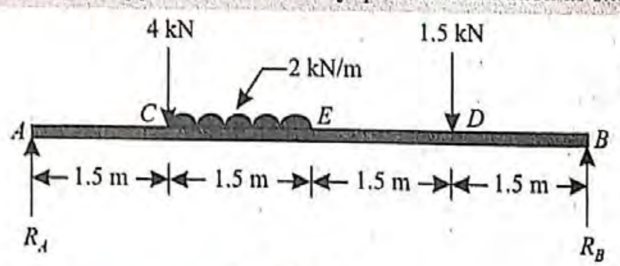


Fig. 9.8.

Determine the reactions R_A and R_B of the beam.

Solution. Given: Span (l) = 6m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve it analytically only.

We know that anticlockwise moment due to the reaction R_B about A.

$$= R_B \times l = R_B \times 6 = 6\,R_B \text{ kN-m} \quad ...(i)$$

and sum* of the clockwise moments about A

$$= (4 \times 1.5) + (2 \times 1.5)\,2.25 + (1.5 \times 4.5) = 19.5 \text{ kN-m} \quad ...(ii)$$

Equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$6\,R_B = 19.5$$

or $$R_B = \frac{19.5}{6} = 3.25 \text{ k.N} \quad \textbf{Ans.}$$

and $$R_A = 4 + (2 \times 1.5) + 1.5 - 3.25 = 5.25 \text{ kN} \quad \textbf{Ans.}$$

Example 9.3. *A simply supported beam AB of span 4.5 m is loaded as shown in Fig. 9.9.*

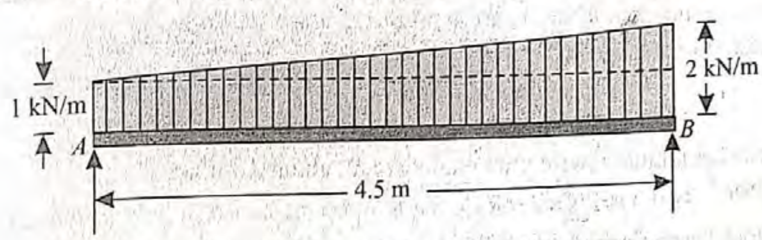


Fig. 9.9.

Find the support reactions at A and B.

Solution. Given: Span (l) = 4.5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

For the sake of simplicity, we shall assume the uniformly varying load to be split†† up into (*a*) a uniformly distributed load of 1 kN/m over the entire span, and (*b*) triangular load of 0 at A to 1 kN/m at B.

We know that anticlockwise moment due to R_B about A

$$= R_B \times l = R_B \times 4.5 = 4.5\,R_B \text{ kN-m} \quad ...(i)$$

and sum of clockwise moments due to uniformly varying load about A

$$= (1 \times 4.5 \times 2.25) + (2.25 \times 3) = 16.875 \text{ kN-m} \quad ...(ii)$$

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$4.5\,R_B = 16.875$$

or $$R_B = \frac{16.875}{4.5} = 3.75 \text{ kN} \quad \textbf{Ans.}$$

and $$R_A = [1 \times 4.5] + \left[4.5 \times \frac{0+1}{2}\right] - 3.75 = 3.0 \text{ kN} \quad \textbf{Ans.}$$

* The uniformly distributed load of 2 kN/m for a length of 1.5 m (*i.e.*, between C and E) is assumed as an equivalent point load of $2 \times 1.5 = 3$ kN and acting at the centre of gravity of the load *i.e.*, at a distance of $1.5 + 0.75 = 2.25$ m from A.

†† The uniformly distributed load of 1 kN/m over the entire span is assumed as an equivalent point load of $1 \times 4.5 = 4.5$ kN and acting at the centre of gravity of the load *i.e.* at a distance of 2.25 m from A. Similarly, the triangular load in assumed as an equivalent point load of $4.5 \times \frac{0+1}{2} = 2.25$ kN and acting at the centre of gravity of the load *i.e.*, distance of $4.5 \times \frac{2}{3} = 3$ m from A.

Example 9.4. *A simply supported beam AB of 6 m span is subjected to loading as shown in Fig. 9.10.*

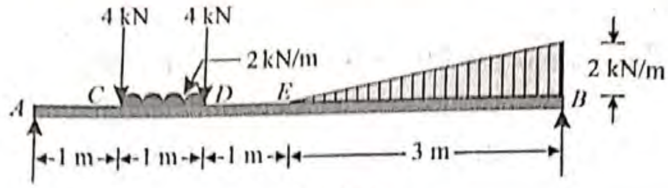


Fig. 9.10.

Find graphically or otherwise, the support reactions at A and B.

Solution. Given: Span (l) = 6 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that anticlockwise moment due to R_B about A

$$= R_B \times l = R_B \times 6 = 6\,R_B \text{ kN-m} \quad ...(i)$$

and *sum of clockwise moments due to loads about A

$$= (4 \times 1) + (2 \times 1)\,1.5 + (4 \times 2) + \frac{(0+2)}{2} \times 3 \times 5 = 30 \text{ kN-m} \quad ...(ii)$$

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$6\,R_B = 30$$

or $$R_B = \frac{30}{6} = 5 \text{ kN} \quad \textbf{Ans.}$$

and $$R_A = (4 + 2 + 4 + 3) - 5 = 8 \text{ kN} \quad \textbf{Ans.}$$

9.13. Overhanging Beams

A beam having its end portion (or portions) extended in the form of a cantilever, beyond its support, as shown in Fig. 9.11 is known as an overhanging beam.

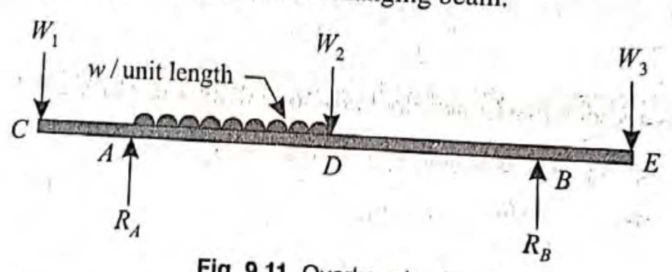


Fig. 9.11. Overhanging beam.

It may be noted that a beam may be overhanging on one of its sides or both the sides. In such cases, the reactions at both the supports will be vertical as shown in the figure.

* It means converting the uniformly distributed load between C and D as well as triangular load between E and B into vertical loads as discussed below:

1. The uniformly distributed load is assumed as an equivalent point load of $2 \times 1 = 2$ kN acting at the centre of gravity of the load *i.e.*, at the mid point of C and D.
2. The triangular load is assumed as an equivalent point load of $\frac{0+2}{2} \times 3 = 3$ kN acting at the centre of gravity of the load *i.e.* at a distance of $\frac{2}{3} \times 3 = 2$ m from E or 5 m from A.

Example 9.5. *A beam AB of span 3m, overhanging on both sides is loaded as shown in Fig. 9.12.*

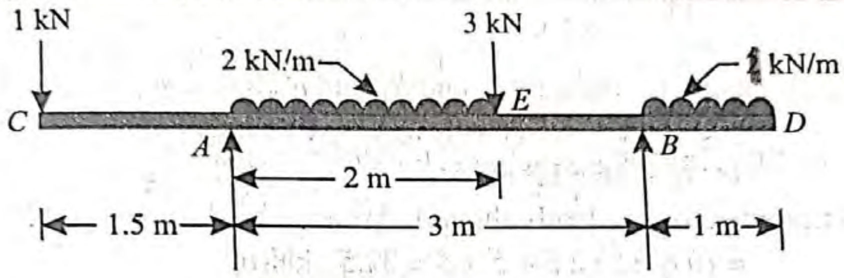


Fig. 9.12.

Determine the reactions at the supports A and B.

Solution. Given: Span $(l) = 3$ m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that anticlockwise moment due to R_B and load* at C about A

$$= R_B \times l + (1 \times 1.5) = R_B \times 3 + (1 \times 1.5) = 3R_B + 1.5 \text{ kN} \quad ...(i)$$

and sum of clockwise moments due to loads about A

$$= (2 \times 2)\ 1 + (3 \times 2) + (1 \times 1)\ 3.5 = 13.5 \text{ kN-m} \quad ...(ii)$$

Now equating anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$3R_B + 1.5 = 13.5$$

or $$R_B = \frac{13.5 - 1.5}{3} = \frac{12}{3} = 4 \text{ kN} \quad \textbf{Ans.}$$

and $$R_A = 1 + (2 \times 2) + 3 + (1 \times 1) - 4 = 5 \text{ kN} \quad \textbf{Ans.}$$

4. A beam AB 6 m long rests on two supports 4 m apart, the right hand end is overhanging by 2 m. The beam carries a uniformly distributed load of 1 kN/m over the entire length of the beam.

Determine the reactions at the two supports.

[Ans. R_A = 1.5 kN, R_B = 4.5 kN]

5. A beam $ABCDEF$ of 7.5 m long and span 4.5 m is supported at B and E. The beam is loaded as shown in Fig. 9.15.

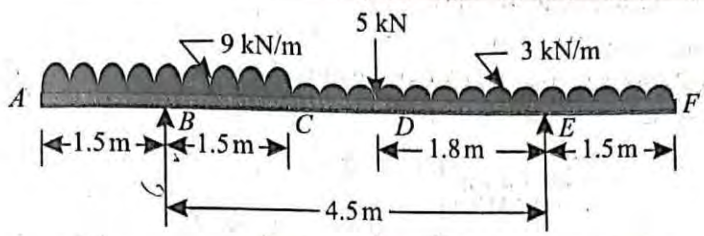


Fig. 9.15.

Find graphically, or otherwise, the support reactions at the two supports.

[Ans. R_B = 29.33 kN, R_E = 12.57 kN]

6. A beam $ABCDE$ hinged at A and supported on rollers at D, is loaded as shown in Fig. 9.16.

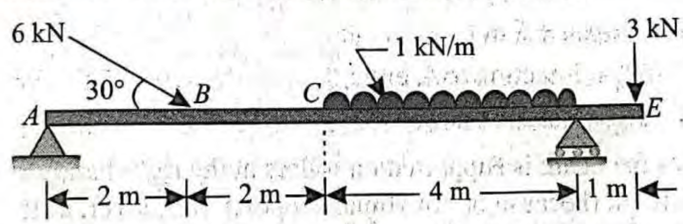


Fig. 9.16.

Find the reactions at A and D. **[Ans.** R_A = 5.94 kN, R_D = 7.125 kN, θ = 61°]

9.14. Roller Supported Beams

In such a case, the end of a beam is supported on rollers, and the reaction on such an end is always *normal to the support,* as shown in Fig. 9.17 (*a*) and (*b*). All the steel trusses, of the bridges, have one of their ends as supported on rollers.

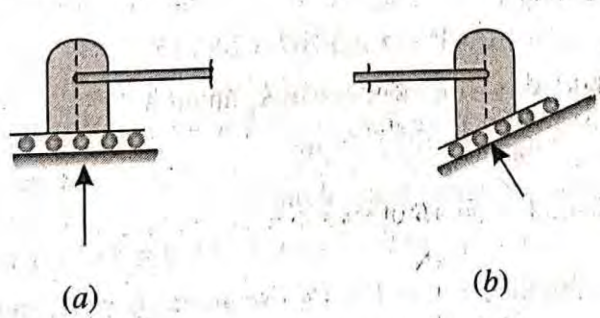


Fig. 9.17. Roller supported end

The main advantage, of such a support, is that the beam can move easily towards left or right, on account of expansion or contraction due to change in temperature.

9.15. Hinged Beams

In such a case, the end of a beam is hinged to the support as shown in Fig. 9.18. The reaction on such an end may be *horizontal*, *vertical* or *inclined*, depending upon the type of loading. All the steel trusses of the bridges have one of their end roller supported, and the other hinged.

Fig. 9.18. Hinged end.

The main advantage of such a support is that the beam remains stable. A little consideration will show, that the beam cannot be stable, if both of its ends are supported on rollers. It is thus obvious, that one of the supports is made roller supported and the other hinged.

Example 9.7. *A beam AB of 6 m span is loaded as shown in Fig. 9.19.*

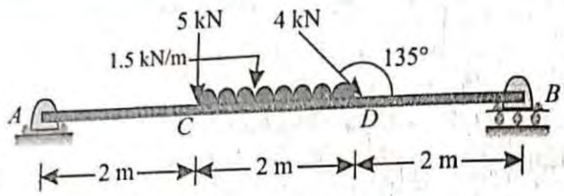


Fig. 9.19.

Determine the reactions at A and B.

Solution. Given: Span = 6 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that as the beam is supported on rollers at the right hand support (B), therefore the reaction R_B will be vertical (because of horizontal support). Moreover, as the beam is hinged at the left support (A) and it is also carrying inclined load, therefore the reaction at this end will be the resultant of horizontal and vertical forces, and thus will be inclined with the vertical.

The example may be solved either analytically or graphically, but we shall solve it by both the methods, one by one.

Analytical method

Resolving the 4 kN load at D vertically

$$= 4 \sin 45° = 4 \times 0.707 = 2.83 \text{ kN}$$

and now resolving it horizontally

$$= 4 \cos 45° = 4 \times 0.707 = 2.83 \text{ kN}$$

We know that anticlockwise moment due to R_B about A

$$= R_B \times 6 = 6\, R_B \text{ kN-m} \quad ...(i)$$

and *sum of clockwise moments due to loads about A

$$= (5 \times 2) + (1.5 \times 2)\, 3 + 2.83 \times 4 = 30.3 \text{ kN-m} \quad ...(ii)$$

Now equating the anticlockwise and clockwise moments in (*i*) and (*ii*),

$$6\, R_B = 30.3$$

or $$R_B = \frac{30.3}{6} = 5.05 \text{ kN} \quad \textbf{Ans.}$$

* Moment of horizontal component of 2.83 kN at D about A will be zero.

We know that vertical component of the reaction R_A

$$= [5 + (1.5 \times 2) + 2.83] - 5.05 = 5.78 \text{ kN}$$

$\therefore$ Reaction at A,

$$R_A = \sqrt{(5.78)^2 + (2.83)^2} = 6.44 \text{ kN} \quad \textbf{Ans.}$$

Let θ = Angle, which the reaction at A makes with vertical.

$$\therefore \quad \tan\theta = \frac{2.83}{5.78} = 0.4896 \quad \text{or} \quad \theta = 26.1° \quad \textbf{Ans.}$$

9.16. Beams Subjected to a Moment

Sometimes, a beam is subjected to a clockwise or anticlockwise moment alongwith loads. In such a case, magnitude of the moment is taken into consideration while calculating the reactions. Since the moment does not involve any load, therefore it has no horizontal or vertical components.

Example 9.10. *Fig. 9.25 shows as beam ABCD simply supported on a hinged support at A and at D on a roller support inclined at 45° with the vertical.*

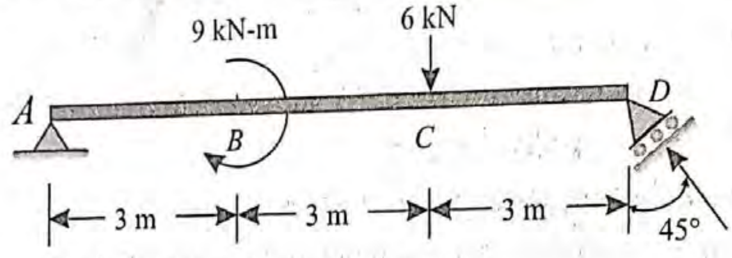


Fig. 9.25.

Determine the horizontal and vertical components of reaction at support A. Show clearly the direction as well as the magnitude of the resultant reaction at A.

Solution. Given: Span = 9 m

Let R_A = Reaction at A, and

R_D = Reaction at D.

The reaction R_D is inclined at 45° with the vertical as given in the example. We know that as the beam is hinged at A, therefore the reaction at this end will be the resultant of vertical and horizontal forces, and thus will be inclined with the vertical.

We know that vertical component of reaction R_D

$$= R_D \cos 45° = R_D \times 0.707 = 0.707\, R_D$$

and anticlockwise moment due to the vertical component of reaction R_D about A

$$= 0.707\, R_D \times 9 = 6.363\, R_D \quad ...(i)$$

We also know that sum of clockwise moments due to moment at B and Load at C about A.

$$= 9 + (6 \times 6) = 45 \text{ kN-m} \quad ...(ii)$$

Now equating the anticlockwise and clockwise moments given in (*i*) and (*ii*),

$$6.363\, R_D = 45$$

or

$$R_D = \frac{45}{6.363} = 7.07 \text{ kN}$$

∴ Vertical component of reaction R_D

$$= 7.07 \cos 45° = 7.07 \times 0.707 = 5 \text{ kN}$$

and horizontal component of R_D (this is also equal to horizontal component of reaction R_A as there is no inclined load on the beam)

$$= 7.07 \sin 45° = 7.07 \times 0.707 = 5 \text{ kN}$$

∴ Vertical component of reaction R_A

$$= 6 - 5 = 1 \text{ kN}$$

and $$R_A = \sqrt{(5)^2 + (1)^2} = 5.1 \text{ kN} \quad \textbf{Ans.}$$

Let θ = Angle, which the reaction at A makes with the vertical.

∴ $$\tan\theta = \frac{5}{1} = 5.0 \quad \text{or} \quad \theta = 78.7° \quad \textbf{Ans.}$$

9.17. Reactions of a Frame or a Truss

A frame or a truss may be defined as a structure made up of several bars, riveted or welded together. The support reactions at the two ends of a frame may be found out by the same principles as those for a beam, and by any one of the following methods:

1. Analytical method, and 2. Graphical method.

9.18. Types of End Supports of Frames

Like the end supports of a beam, frames may also have the following types of supports :

1. Frames with simply supported ends.
2. Frames with one end hinged and the other supported freely on rollers.
3. Frames with both the ends fixed.

9.19. Frames with Simply Supported Ends

It is a theoretical case in which the ends of a frame are simply supported. In such a case, both the reactions are always vertical and may be found out by the principle of moments *i.e.* by equating the anticlockwise moments and clockwise moments about one of the supports.

Example 9.11. *A truss of 9 m span is loaded as shown in Fig. 9.26.*

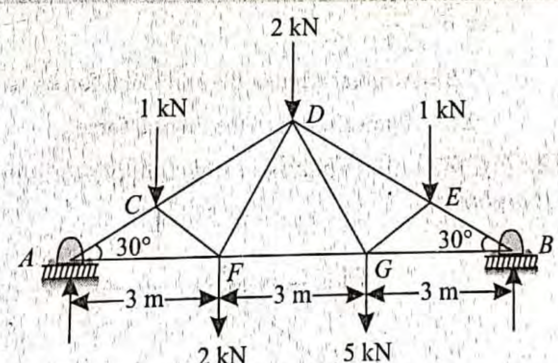


Fig. 9.26.

Find the reactions at the two supports.

Solution. Given: Span $AB = 9$ m

Let R_A = Reaction at A, and

R_B = Reaction at B.

From the geometry of the figure, we know that perpendicular distance between A and the lines of action of the loads at C, D and E are 2.25m, 4.5 m and 6.75 m respectively.

Now equating the anticlockwise and clockwise moments about A,

$$R_B \times 9 = (1 \times 2.25) + (2 \times 4.5) + (1 \times 6.75) + (2 \times 3) + (5 \times 6) = 54 \text{ kN-m}$$

∴ $$R_B = \frac{54}{9} = 6.0 \text{ kN} \quad \textbf{Ans.}$$

and $$R_A = (1 + 2 + 1 + 2 + 5) - 6.0 = 5.0 \text{ kN} \quad \textbf{Ans.}$$

9.20. Frames with one End Hinged (or Pin-jointed) and the Other Supported Freely on Rollers

Sometimes, a frame is hinged (or pin-jointed) at one end, and freely supported on rollers at the other end. If such a frame carries vertical loads only, the problem does not present any special features. Such a problem may be solved just as a simply supported frame.

But sometimes such a frame carries horizontal or inclined loads (with or without vertical loads). In such a case, the support reaction at the roller supported end will be normal to the support. The support reaction at the hinged end will be the resultant of :

1. Vertical reaction, which may be found out by subtracting the vertical component of the support reaction at the roller supported end from the total vertical loads.
2. Horizontal reaction, which may be found out by algebraically adding all the horizontal loads.

Now we shall discuss the following types of loadings on frames with one end hinged (or pin-jointed) and other supported on rollers.

1. Frames carrying horizontal loads, and
2. Frames carrying inclined loads.

9.21. Frames with One End Hinged (or Pin-jointed) and the Other Supported on Rollers and Carrying Horizontal Loads

We have already discussed in the last article that the support reaction at the roller supported end will be normal to the support. The support reaction at the hinged end will be the resultant of vertical and horizontal forces.

Note: The inclination of the resultant reaction (θ) with the vertical is given by the relation :

$$\tan\theta = \frac{\Sigma H}{\Sigma V}$$

where ΣH = Algebraic sum of the horizontal forces, and

ΣV = Algebraic sum of the vertical forces.

Example 9.12. *Fig. 9.27 shows a framed structure of 4 m span and 1.5 m height subjected to two point loads at B and D.*

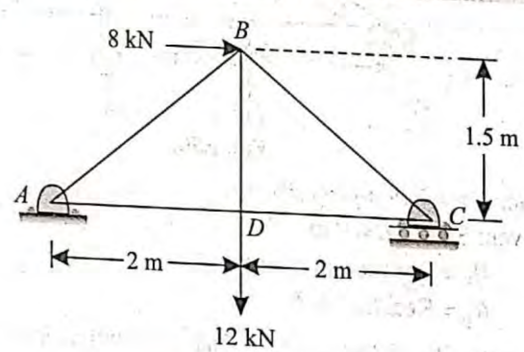


Fig. 9.27.

Find graphically or otherwise the reactions at A and C.

Solution. Given: Span = 4 m

Let R_A = Reaction at A, and

R_C = Reaction at C

Since the structure is supported on rollers at the right hand support (C), therefore the reaction at this support will be vertical (because of horizontal support). The reaction at the left hand support (A) will be the resultant of vertical and horizontal forces, and thus will be inclined with the vertical. Taking moments about A and equating the same,

$$R_C \times 4 = (8 \times 1.5) + (12 \times 2) = 36$$

$$\therefore \quad R_C = V_C = \frac{36}{4} = 9.0 \text{ kN} \qquad \textbf{Ans.}$$

Now vertical component of reaction R_A

$$V_A = 12 - 9 = 3 \text{ kN}$$

and horizontal reaction at the left hand support A,

$$H_A = 8 \text{ kN} \quad (\leftarrow)$$

$\therefore$ Reaction at A, $\quad R_A = \sqrt{(8)^2 + (3)^2} = 8.54 \text{ kN} \qquad$ **Ans.**

Let $\quad \theta$ = Angle, which the reaction R_A makes with the vertical.

$$\therefore \quad \tan\theta = \frac{8}{3} = 2.6667 \quad \text{or} \quad \theta = 69.4° \qquad \textbf{Ans.}$$

9.22. Frames with one End Hinged (or Pin-jointed) and the Other Supported on Rollers and Carrying Inclined Loads

We have already discussed in Art. 9.20 that the support reaction at the roller supported end will be normal to the support. And the support reaction at the hinged end will be the resultant of vertical and horizontal forces. The support reactions for such a frame may be found out by the following methods :

1. Analytical method.
2. Graphical method

Example 9.15. *Fig. 9.30 shows a roof truss hinged at one end and rests on rollers at the other. It carries wind loads as shown in the figure.*

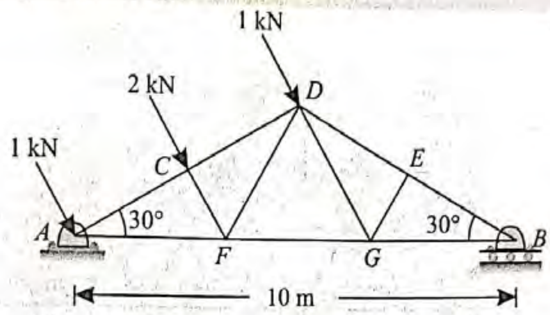


Fig. 9.30.

Determine graphically, or otherwise, the reactions at the two supports.

Solution. Given: Span = 10 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that as the roof truss is supported on rollers at the right hand support (B), therefore the reaction at this end will be vertical (because of horizontal support). Moreover, as truss is hinged at the left support (A) and is also carrying inclined loads, therefore the reaction at this end will be the resultant of horizontal and vertical forces, and thus will be inclined with the vertical.

The example may be solved either analytically or graphically. But we shall solve it by both the methods one by one.

Analytical Method

From the geometry of the figure, we find that perpendicular distance between the support A and the line of action of the load at D.

$$= \frac{5}{\cos 30°} = \frac{5}{0.866} = 5.8\,\text{m}$$

and perpendicular distance between the support A and the line of action of the load at C.

$$= \frac{5.8}{2} = 2.9\ \text{m}$$

Now equating the anticlockwise moments and clockwise moments about A,

$$R_B \times 10 = (2 \times 2.9) + (1 \times 5.8) = 11.6$$

$$\therefore \quad R_B = \frac{11.6}{10} = 1.16\ \text{kN} \qquad \textbf{Ans.}$$

We know that total wind load

$$= 1 + 2 + 1 = 4\ \text{kN}$$

$\therefore$ Horizontal component of the total wind load

$$= 4 \cos 60° = 4 \times 0.5 = 2\ \text{kN}$$

and vertical component of the total wind load

$$= 4 \sin 60° = 4 \times 0.866 = 3.464 \text{ kN}$$

∴ Balance vertical reaction at A

$$= 3.464 - 1.16 = 2.304 \text{ kN}$$

and reaction at A, $R_A = \sqrt{(2)^2 + (2.304)^2} = 3.05 \text{ kN}$

Let θ = Angle, which the reaction R_A makes with the vertical.

∴ $\tan \theta = \dfrac{2.0}{2.304} = 0.868$ or $\theta = 41°$ **Ans.**

9.23. Frames with Both Ends Fixed

Sometimes, a frame or a truss is fixed or built-in at its both ends. In such a case, the reactions at both the supports cannot be determined, unless some assumption is made. The assumptions, usually, made are:

1. The reactions are parallel to the direction of the loads, and
2. In case of inclined loads, the horizontal thrust is equally shared by the two reactions.

Generally, the first assumption is made and the reactions are determined, as usual, by taking moments about one of the supports.

Example 9.18. *Fig. 9.35 shows a roof truss with both ends fixed. The truss is subjected to wind loads, normal to the main rafter as shown in the figure.*

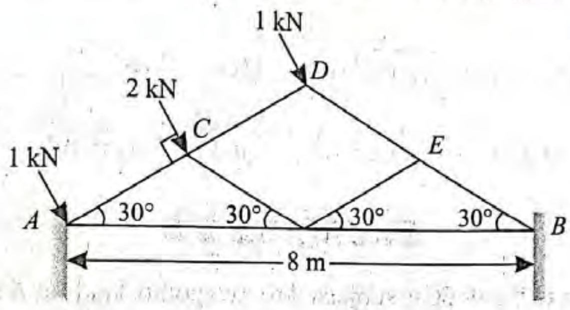


Fig. 9.35.

Find the reactions at the supports.

Solution. Given: Span of truss = 8 m

Let R_A = Reaction at the left support A, and

R_B = Reaction at the right support B.

This example may be solved by any one of the two assumptions as mentioned in Art. 9.23. But we shall solve it by both the assumptions, one by one.

Assuming that the reactions are parallel to the direction of the loads.

Equating the anticlockwise and clockwise moments about A,

$$R_B \times 8 \sin 60° = \frac{2 \times 2}{\cos 30°} + \frac{1 \times 4}{\cos 30°} = \frac{8}{0.866} = 9.24$$

$$\therefore \quad R_B = \frac{9.24}{8 \sin 60°} = \frac{9.24}{8 \times 0.866} = 1.33 \text{ kN}$$

and $$R_A = (1 + 2 + 1) - 1.33 = 2.67 \text{ kN} \quad \textbf{Ans.}$$

Assuming that the horizontal thrust is equally shared by two reactions

Total horizontal component of the loads,

$$\Sigma H = 1 \cos 60° + 2 \cos 60° + 1 \cos 60° \text{ kN}$$
$$= (1 \times 0.5) + (2 \times 0.5) + (1 \times 0.5) = 2 \text{ kN}$$

$\therefore$ Horizontal thrust on each support,

$$R_{AH} = R_{BH} = \frac{2}{2} = 1 \text{kN}$$

Now equating the anticlockwise and clockwise moments about A,

$$R_{BV} \times 8 = \frac{2 \times 2}{\cos 30°} + \frac{1 \times 4}{\cos 30°} = \frac{8}{0.866} = 9.24$$

$\therefore$ $$R_{BV} = \frac{9.24}{8} = 1.15 \text{ kN}$$

and $$R_{AV} = (1 \sin 60° + 2 \sin 60° + 1 \sin 60°) - 1.15 \text{ kN}$$
$$= (1 \times 0.866 + 2 \times 0.866 + 1 \times 0.866) - 1.15 = 2.31$$

$\therefore$ Reaction at A,

$$R_A = \sqrt{(1)^2 + (2.31)^2} = 2.52 \text{ kN} \quad \textbf{Ans.}$$

and $$\tan \theta_A = \frac{2.31}{1} = 2.31 \quad \text{or} \quad \theta_A = 66.6° \quad \textbf{Ans.}$$

Similarly, $$R_B = \sqrt{(1)^2 + (1.15)^2} = 1.52 \text{ kN} \quad \textbf{Ans.}$$

and $$\tan \theta_B = \frac{1.15}{1} = 1.15 \quad \text{or} \quad \theta_B = 49° \quad \textbf{Ans.}$$

EXERCISE 9.2

1. A truss shown in Fig. 9.36 is subjected to two points loads at B and F.

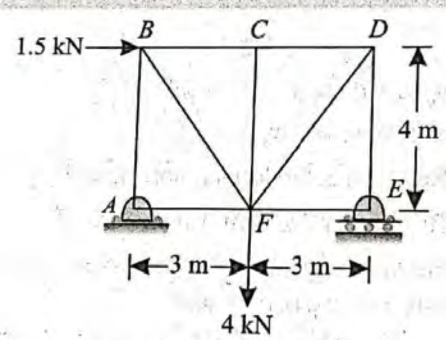


Fig. 9.36.

Find by any method the reactions at A and E.

[Ans. R_A = 1.8 kN, R_E = 3.0 kN, θ = 56.3°]

2. A truss is subjected to two point loads at A as shown in Fig. 9.37.

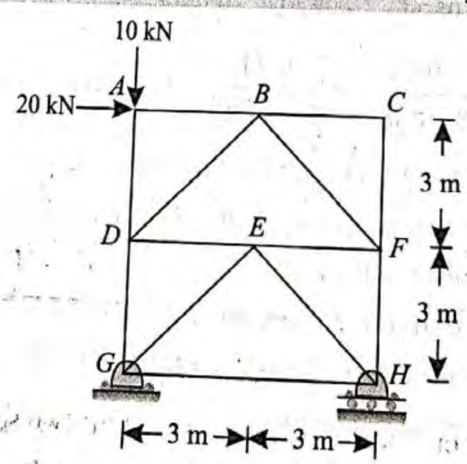


Fig. 9.37.

Find the reactions at G and H. **[Ans.** R_G = 22.4 kN, R_H = 20 kN, θ = 63.4°]